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A wavelet approach to fault diagnosis of a gearbox under varying load conditions

Xiyang Wang [a], Viliam Makis [b,*], Ming Yang [b]

[a] Department of Mechanical Engineering, Nanchang Hangkong University, 330062 Nanchang, Jiangxi, People's Republic of China
[b] Department of Mechanical and Industrial Engineering, University of Toronto, 5 King's College Road, Toronto, Ont., Canada M5S 3G8

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ABSTRACT

Varying load can cause changes in a measured gearbox vibration signal. However, conventional techniques for fault diagnosis are based on the assumption that changes in vibration signal are only caused by deterioration of the gearbox. There is a need to develop a technique to provide accurate state indicator of gearbox under fluctuating load conditions. This paper presents an approach to gear fault diagnosis based on complex Morlet continuous wavelet transform under this condition. Gear motion residual signal, which represents the departure of time synchronously averaged signal from the average tooth-meshing vibration, is analyzed as source data due to its lower sensitiveness to the alternating load condition. A fault growth parameter based on the amplitude of wavelet transform is proposed to evaluate gear fault advancement quantitatively. We found that this parameter is insensitive to varying load and can correctly indicate early gear fault. For a comparison, the advantages and disadvantages of other measures such as kurtosis, mean, variance, form factor and crest factor, both of residual signal and mean amplitude of continuous wavelet transform waveform, are also discussed. The effectiveness of the proposed fault indicator is demonstrated using a full lifetime vibration data history obtained under sinusoidal varying load.

1. Introduction

Fault diagnosis of gear transmission systems has attracted increasing interest in recent years, due to the need to decrease the downtime on production machinery and to reduce the extent of the secondary damage caused by failures. Typically, gearbox diagnostic algorithms use vibration data collected from accelerometers located on the transmission housing. The vibration signals collected by these sensors tend to be composites of vibrations associated with all transmission components, such as the meshing gears, shafts, bearings, and other parts.

Gearboxes often operate under fluctuating load conditions during service. The fault diagnosis technique becomes more complicated when gearbox is subject to varying operating condition. In most situations, varying operating condition means fluctuating load condition or torque level since it is the major source of contribution to the energy of the measured vibration signal. In manufacturing processes, many unexpected sources can contribute to the fluctuation of load condition, e.g. nonhomogeneity of raw material in the machining process causing variation of load applied on tooling system and then transmitting the varying load condition back to the output shaft of a gearing system [1]. The cutting-head gearbox of a

* Corresponding author. Tel.: +1 4169784184; fax: +1 4169783453.
E-mail addresses: wxywzh@yahoo.com.cn (X. Wang), makis@mie.utoronto.ca (V. Makis), yangming@mie.utoronto.com (M. Yang).

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continuous miner and the drag gearboxes of a dragline are typical examples of large, expensive gearboxes that operate under fluctuating load conditions during service.

Conventional vibration-monitoring techniques are based on the assumption that changes in the measured structural response are caused by deterioration in the condition of the gearbox. However, fluctuating load condition can also cause changes of measured gearbox vibration signal. Most of the vibration-monitoring techniques available cannot be used for the diagnosis of gearbox under this condition, since they are based on the assumption that the load is constant.

Some researchers have focused on this problem. Randall [2] states that the amplitude of the vibration of the gearbox casing, caused by the meshing of the gears, is modulated by the fluctuation in the torque load. McFadden [3,4] mentions that changing load conditions influenced the results he obtained for time domain synchronized averaged vibration. Stander and Heyns [5,6] considered various kinds of faults and several load conditions and found that Mahalanobis distance could be used as a single parameter to indicate the progression of faults under fluctuating load condition. The procedure was tested using experimental data collected under constant, sinusoidal, step and chirp-load fluctuations for different levels of damage severity. Statistical parameters were obtained from the pseudo-Wigner–Ville distributions, which had been calculated for the load-normalized acceleration signals averaged in the rotation domain. These parameters were linearly separated according to the fault severity levels under different load conditions. It was indicated that the Mahalanobis distance could be utilized to combine the parameters into a single-value parameter, which can be monotonically trended to indicate the progression of fault severity. However, this method was validated using only limited data obtained at a few specific inspection times. The full lifetime behavior using this method was not investigated.

Wang and Wong [7] developed a linear prediction method, which is based on the assumption that the vibration caused by a healthy pair of gears can be modeled as a stationary autoregressive process. As such a process is unable to represent the non-stationary transients associated with the localized faults in gear teeth, the error in the prediction increases when localized defects occur. The authors stated that the approach is independent of operational conditions, but the quantitative indicators for fault advancement were not developed in the paper. Baydar and Ball [8] used a time–frequency distribution called instantaneous power spectrum (IPS) to detect local faults on the teeth of a gear under different nominal load conditions. It was shown that the IPS can be used to detect local faults not only under constant speed and load, but it can also indicate the progression of faults under varying load conditions, whereas the conventional spectrum analysis can be misleading as a diagnostic tool when it is used under varying load conditions.

Zhan et al. [1] proposed a technique for state detection of a gearbox based on fitting a time-varying autoregressive model to the gear motion residual signals by applying a noise-adaptive Kalman filter in the healthy state of the target gear. The optimum autoregressive model order was determined for a compromised model fitted to the healthy gear motion residual signals collected under various load conditions applying a specific model order selection method proposed in that paper.

No further information could be found in the literature on the signal processing and diagnosis of gearboxes under varying load conditions. Wavelet transform (WT), which is capable of providing both the time-domain and frequency-domain information simultaneously, has been successfully used in non-stationary vibration signal processing and fault diagnosis [9]. Kurtosis has also been used frequently in engineering signal processing applications for detection of fault symptoms because it is sensitive to sharp changes in signal behavior [10]. In this paper, the time synchronously averaged (TSA) residual signals are used as source signals for analysis due to their lower sensitivity to the alternating load, as suggested by Zhan et al. [1]. An approach which uses the continuous wavelet transform (CWT) of the residual signal is presented in this paper. The properties of kurtosis and peak value of the mean amplitude of continuous wavelet transform coefficient as quantitative indicators of gear fault advancement over full lifetime of gearbox are explored. It is shown that these indicators are insensitive to varying load.

The remainder of the paper is organized as follows. In Section 2, we provide a quick overview of the continuous wavelet transform and focus on parameter selection of a complex continuous wavelet transform function. Fault feature extraction functions based on continuous wavelet transform amplitude are then considered in Section 3. In Section 4, we describe briefly the experimental gear test rig, and summarize the data-processing techniques used in this study. Fault diagnosis processes under sinusoidal varying load condition using the indicators proposed in Section 3 are analyzed afterwards in Section 5, and the effect of varying load on these indicators is described.

2. Review of wavelet transform

Wavelet transform provides a good means of studying how the frequency content changes with time and consequently is able to detect and localize short-duration phenomena. Wavelet coefficients measure the similarity between the signal and each of its daughter wavelets. The more the daughter wavelet is similar to the feature component, the larger is the corresponding wavelet coefficient. The basic principle is to choose a wavelet function whose shape is similar to the vibration signal caused by the mechanical fault. When a fault occurs, the vibration signal of the machine which includes periodic impulses is well described by Morlet wavelet which has been widely used in fault diagnosis. The complex-valued Morlet wavelet function is defined as [11]:

$$\psi(t) = (\pi f_b)^{-0.5}\, e^{j2\pi f_c t}\, e^{-t^2/f_b}, \tag{1}$$

Table 1
Specification of test run #14.

Gearbox ID#	DS3S034014	Number of teeth (Drive)	70
Make	Dodge APG	Number of teeth (Pinion)	21
Model	R86005	Maximum rated input hp	4.66
Rated input speed	1750 rpm	Gear ratio	3.333
Maximum rated output torque	62.73 N-M	Gear mesh frequency (Hz)	613

The table is quoted from the MDTB data CDs TR#14.

where f_b is the bandwidth parameter and f_c is the wavelet center frequency. A remarkable feature of this wavelet is that its Fourier spectrum is Gaussian function:

$$\Psi(f) = \mathrm{e}^{-\pi^2 f_b (f - f_c)^2}. \tag{2}$$

Choosing a suitable combination of bandwidth and center frequency parameters is a design question, and depends on the signal under analysis. Wang [12] described a method to choose these parameters based on the sampling frequency and the number of sampling points of a vibration signal, which is suitable for on-line monitoring and automatic fault diagnosis. In practice, f_b and f_c are often determined experimentally based on the analyzed frequency range and required resolution of signals in both time and frequency domain. The min and max central frequency f_c of every shifted wavelet should cover the whole bandwidth of analyzing frequency. The optimal bandwidth parameter f_b is the one that leads to minimal value of the wavelet entropy. In this paper, the values of f_b and f_c were determined experimentally. Several bandwidth and center frequency parameters (from cmor 0.5-1 to cmor 70-1) were tested and compared, the best results were obtained for cmor 0.5-1 which gave the minimal value of the wavelet entropy and best resolution in the time and frequency domain (Table 1).

Continuous wavelet transform provides both amplitude and phase map. Although the phase map has been proved to display distinctive features in the presence of a cracked tooth [13], only the amplitude will be considered and represented using a linear scale in this paper, because it is difficult to obtain a quantitative parameter from a phase map.

3. Fault feature extraction using continuous wavelet transform

The signal processed by wavelet transform can be the raw vibration signal, the time synchronously averaged signal or the gear motion residual signal. Gear motion residual signal represents the departure of time synchronously averaged signal from the average tooth-meshing vibration and usually shows evidence of faults earlier and more clearly than time synchronously averaged signal. Since the majority of energy in the healthy state of the target gear is concentrated at the meshing fundamental and its harmonics, the gear motion residual signal will be much less sensitive to the alternating load than the time synchronously averaged signal. The fault-induced non-stationary features will be more significant in the gear motion residual signal, which is usually stationary in the healthy state of the target gear. Therefore, in this paper, the gear motion residual signal under varying load conditions is used to as source signal and the wavelet transform is applied to this signal.

By synchronizing the sampling of the vibration signal with the rotation of a particular gear and evaluating the ensemble average over many revolutions with the start of each frame at the same angular position, a signal called the time synchronously averaged signal is obtained [14], which in practice contains only the components which are synchronous with the revolution of the gear in question. This process strongly reduces the effects of all other sources, including other gears, and the noise. The time synchronously averaged signal can be expressed as [14]:

$$g(t) = \sum_{m=0}^{M} X_m(1 + a_m(t))\cos(2\pi m f_x t + \phi_m + b_m(t)), \tag{3}$$

where M is the number of tooth-meshing harmonics, f_x is the tooth-meshing frequency, X_m and Φ_m are, respectively, the amplitude and the phase of the mth meshing harmonic, while the modulation effects concerning the same harmonic are given by the amplitude modulation function, $1+a_m(t)$, and the phase modulation function, $b_m(t)$. These modulation functions are periodic with the considered gear rotation frequency.

The residual signal $x(t)$ is obtained by eliminating the fundamental and harmonics of the tooth-meshing frequency and ghost components from the FFT spectrum of the time synchronously averaged signal and then reconstructing the remaining signal in the time domain. The eliminated components constitute the "regular" signal, $y(t)$. Thus, the residual signal can be expressed as

$$x(t) = g(t) - y(t). \tag{4}$$

Dalpiaz, Rivola and Rubini [14] compared the results of wavelet transform using the raw signal, time synchronously averaged signal and residual signal of gear motion. The results show that the wavelet transform of the raw signal is practically insensitive to cracks, while the sensitivity of the wavelet transform of time synchronously averaged signal is quite satisfactory. Because the crack effects are evident only in some frequency ranges, a fault detection procedure would be strongly dependent on the choice of a proper cross-section in the wavelet transform map. This limitation is overcome if the residual of the time synchronously averaged signal is processed by the wavelet transform. Boulahbal, Golnaraghi and Ismail [13] used both the amplitude and phase maps of the wavelet to assess the condition of a gear, and also found that the amplitude wavelet map of the residual vibration signal offers a better indicator of the presence of faults than the map of the actual signal. However, in these papers the effects of varying loads were not discussed.

Effective fault diagnosis and prognosis are based on the quality of the selected and extracted fault features. When continuous wavelet transform is used, fault feature extracting techniques can be classified as the wavelet coefficients based, wavelet energy based, singularity based, wavelet function based, etc. [9]. Since the wavelet coefficients will highlight the changes in signals, which often indicate the occurrence of fault, the wavelet coefficients-based features are suitable for early fault detection. However, the slight energy changes in gear motion residual signals caused by gear fault can be easily hidden in the wavelet energy-based features, so the wavelet energy-based features are often not able to detect early faults. The singularity-based features can suffer from the influence of noise, even a slight noise will cause a remarkable change in the singularities.

So, the amplitude of complex continuous wavelet transform of residual signal, $|W_x(a,b;\psi)|$, is used as fault scalogram here. It is a two-dimensional matrix, which has the number of rows and columns equal to the number of the scales and the sampling points, respectively. The value of $|W_x(a,b;\psi)|$ is expected to increase if gear is broken or has the tendency to break. The mean maximum value and mean minimum value of $|W_x(a,b;\psi)|$ over all scales are denoted respectively as:

$$AM_{\max} = \frac{1}{N}\sum_{i=0}^{N} \max_j |W_x(a_i, b_j;\psi)|, \tag{5}$$

$$AM_{\min} = \frac{1}{N}\sum_{i=o}^{N} \min_j |W_x(a_i, b_j;\psi)|, \tag{6}$$

where N is the number of scales which is usually limited and determined by computer capacity, x is the residual signal, a_i is the scale parameter, b_j is the time parameter (sampling points), ψ is the analyzing wavelet, i is the index of scale parameter, j is the index of the time parameter. $a_{\min} = 2^0$ and $a_{\max} = 2^N$ are the minimum and maximum scales in each column of matrix $|W_x(a,b;\psi)|$, respectively. Furthermore, the mean value of $|W_x(a,b;\psi)|$ over all scales and all sampling points is computed as:

$$\overline{AM} = \frac{1}{N}\sum_{i=0}^{N}\left(\frac{1}{M}\sum_{j=1}^{M} |W_x(a_i, b_j;\psi|\right). \tag{7}$$

where M is the number of sampling points which is determined by the original data. Then, the fault-growth parameter (FGP) is defined as:

$$FGP = \frac{AM_{\max} - AM_{\min}}{\overline{AM}}. \tag{8}$$

From our experience, the influence of fluctuating load in residual signal $x(t)$ cannot be completely eliminated if only the fundamental and several harmonics of the tooth-meshing frequency are removed from the FFT spectrum of time synchronously averaged signal. FGP measures the amplitude change relative to the actual amplitude value that is proportional to load, so it is expected to be able to eliminate the influence of varying loads in residual signals.

In order to compare the sensitivity to load and effectiveness to indicate fault occurrence, other statistical measures such as kurtosis, mean, variance, crest factor and form factor, both of the residual signal and of the mean amplitude waveform of continuous wavelet transform, are also computed in the following sections.

4. Experimental set-up

The vibration data used in this paper were obtained from the mechanical diagnostics test-bed in the Applied Research Laboratory at the Pennsylvania State University [15,16]. The mechanical diagnostics test-bed is functionally a motor-drive train-generator test stand. The gearbox is driven at a pre-determined input speed using a 22.05 kW, 1750 rpm AC drive motor, and the torque is applied by a 55.125 kW, 1750 rpm AC absorption motor.

A vector unit capable of controlling the current output of the absorption motor accomplishes the variation of the torque. The mechanical diagnostics test-bed is highly efficient because the electrical power that is generated by the absorber is fed back to the driver motor. The mechanical and electrical losses are sustained by a small fraction of wall power. The mechanical diagnostics test-bed has the capability of testing single and double reduction industrial gearboxes with ratios from about 1.2:1 to 6:1. The gearboxes are nominally in the 3.675–14.7 kW range. The system is designed to provide the maximum versatility to speed and torque settings.

Data was collected in a 10 s window at pre-determined times which cover 200 000 sampling points in total, and triggered by accelerometer RMS thresholds. The sampling frequency is 20 kHz. The signals of the mechanical diagnostics test-bed accelerometers are all converted to digital data format with the highest resolution to which the accelerometers are accurate. As suggested by Miller [17], among all accelerometers located in the mechanical diagnostics test-bed, the single axis shear piezoelectric accelerometer data A03 for axial direction represents the best quality data for state diagnosis of gearbox. Therefore, data recorded by this accelerometer are selected in this study to investigate the proposed continuous wavelet transform indicators for gear fault. The target gear is always the driven gear since its failure is the major factor that caused the test bed shutdown.

Test run #14, which is investigated in this paper, is obtained from the third gearbox in a series of five single reduction helical 1:3.333 ratio gearboxes. It ran at 100% output torque and Hp for 96 h, then the torque and Hp were increased to 300% until five fully broken and two partially broken teeth on the output gear occurred. This run was periodically stopped to allow oil samples and gear photos to be taken. When bringing the run to a stop for these purposes, data was sampled as the load dropped to 250%, 200%, 150%, 100% and 50%. After the oil samples and photos were taken (approx. 15 min), data was sampled as the load was brought back, increased by 50%, 100%, 150%, 200%, and 250%, before reaching 300%. This procedure was performed only during the overload period. In total, 323 data files were collected during this test run. The output torque level was 62.17 N-M before data file 188, and switched to sinusoidally varying afterward.

5. Signal processing and discussion

5.1. Residual signals and their measurement

The time synchronously averaged signals and residual signals of several randomly selected data files obtained under different torque levels and different gear states are shown in Fig. 1. The signals in the plots correspond to one wheel revolution. The following results can be observed from the plots:

(1) The amplitude value of the time synchronously averaged signal is highly sensitive to the torque level. In the same gear state, the higher the torque level, the bigger the amplitude of time synchronously averaged signals (see Figs. 1(a)–(f)). In theory, the varying load has no influence on the residual motion signal, but actually, the amplitude value and waveform of residual signal are also sensitive to the varying load to some extents, especially when the torque level is higher (see Figs. 1(c) and (d)). This observation agrees with the description in Section 3. We can also observe that the influence of varying load is weaker in the residual signal than in the time synchronously averaged signal.
(2) In Figs. 1(h) and (i), evident fault impulses occur both in the time synchronously averaged signal and residual signal, where one gear tooth has already been broken. In Fig. 1(j), there are several greater fault impulses, revealing that several teeth have been broken. The fault impulse in residual signal is more obvious. However, these fault indications are not helpful for early diagnosis and prognosis of gearbox faults in practice. It is hardly possible to find and to evaluate the early gear fault advancement quantitatively only through the waveform of time synchronously averaged signal and residual signal.
(3) There is no evident indication both in the time synchronously averaged signal and residual signal when there is a very small crack in teeth and gear tends to be broken (Fig. 1(g)).

The kurtosis of the residual signal and time synchronously averaged signal over full gearbox lifetime, as well as the varying output torque itself, are plotted in Figs. 1(a)–(c), respectively. Kurtosis is used in engineering for detection of fault symptoms because it is sensitive to impulses in signals. The sharper the impulse in a signal, the greater the kurtosis. The files 119–135 are invalid according to the MDTB report, so the corresponding values of kurtosis are set to be zero. The following results can be drawn from the plots:

(1) The level of kurtosis of residual signal (*KRS*) starting with data file 188 does not vary with the fluctuating torque and remains constant until the teeth are broken, but the influence of varying torque cannot be fully removed from the residual signal (Fig. 2(b)). This can be easily understood because kurtosis is a measure of the “peakedness” of the residual signal waveform and not the actual amplitude value. Higher kurtosis means more variability due to infrequent extreme deviations.
(2) The kurtosis of the time synchronously averaged signal is sensitive to varying load. The kurtosis value starting with data file 188 oscillates with the sinusoidal varying torque level (Fig. 2(c)). The kurtosis variation of the time synchronously averaged signal caused by the actual gear fault is masked by the variation caused by fluctuating torque, so it cannot be used as a fault indicator.
(3) The value of *KRS* before data file 285 is around 3.05, but it increases to 6.13 in data file 285 (see Fig. 2(b)). The value 6.13 in the data file 285 (compared to the value 3.05 before data file 285) is a clear indication of the fault presence. In addition, it is well known that—after the first stage of the fault development—the kurtosis tends to decrease as the fault tends to distribute over more teeth; then, it will increase again for catastrophic faults (several broken teeth). We can add that a dramatic decrease in the kurtosis level can be observed (file 300, Fig. 2(b)) after a sharp increase when several teeth have been broken. Thus, we can conclude that kurtosis value of residual signal is not proportional to the advancement of gear fault, particularly when multiple teeth are involved in a fault.

The mean and variance of residual signal over full gear lifetime are plotted in Figs. 2(d) and (e), respectively. It can be seen that both of these statistics are sensitive to varying loads.

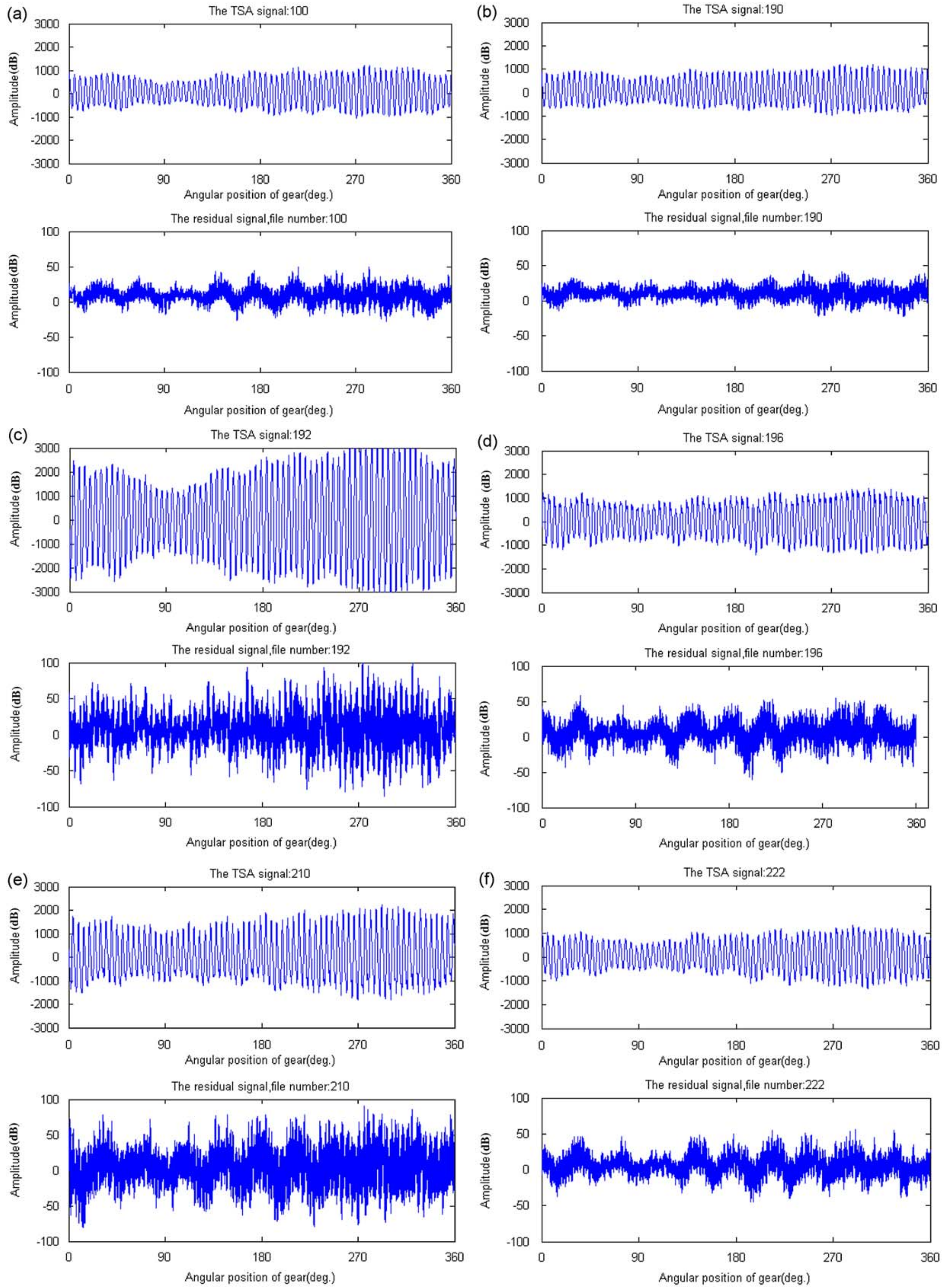

Fig. 1. Time synchronously averaged signal and residual signal: (a) 100% nominal torque level (62.73 N-M), healthy state. (b) 50% nominal torque level (25.66 N-M), healthy state. (c) 300% nominal torque level (184.58 N-M), healthy state. (d) 150% nominal torque level (92.12 N-M), healthy state. (e) 300% nominal torque level (184.46 N-M), healthy state. (f) 200% nominal torque level (123.77 N-M), healthy state. (g) 200% nominal torque level (123.77 N-M), teeth crack. (h) 250% nominal torque level (154.51 N-M), teeth broken. (i) 200% nominal torque level (123.88 N-M), teeth broken. (j) 300% nominal torque level (184.58 N-M), teeth broken.

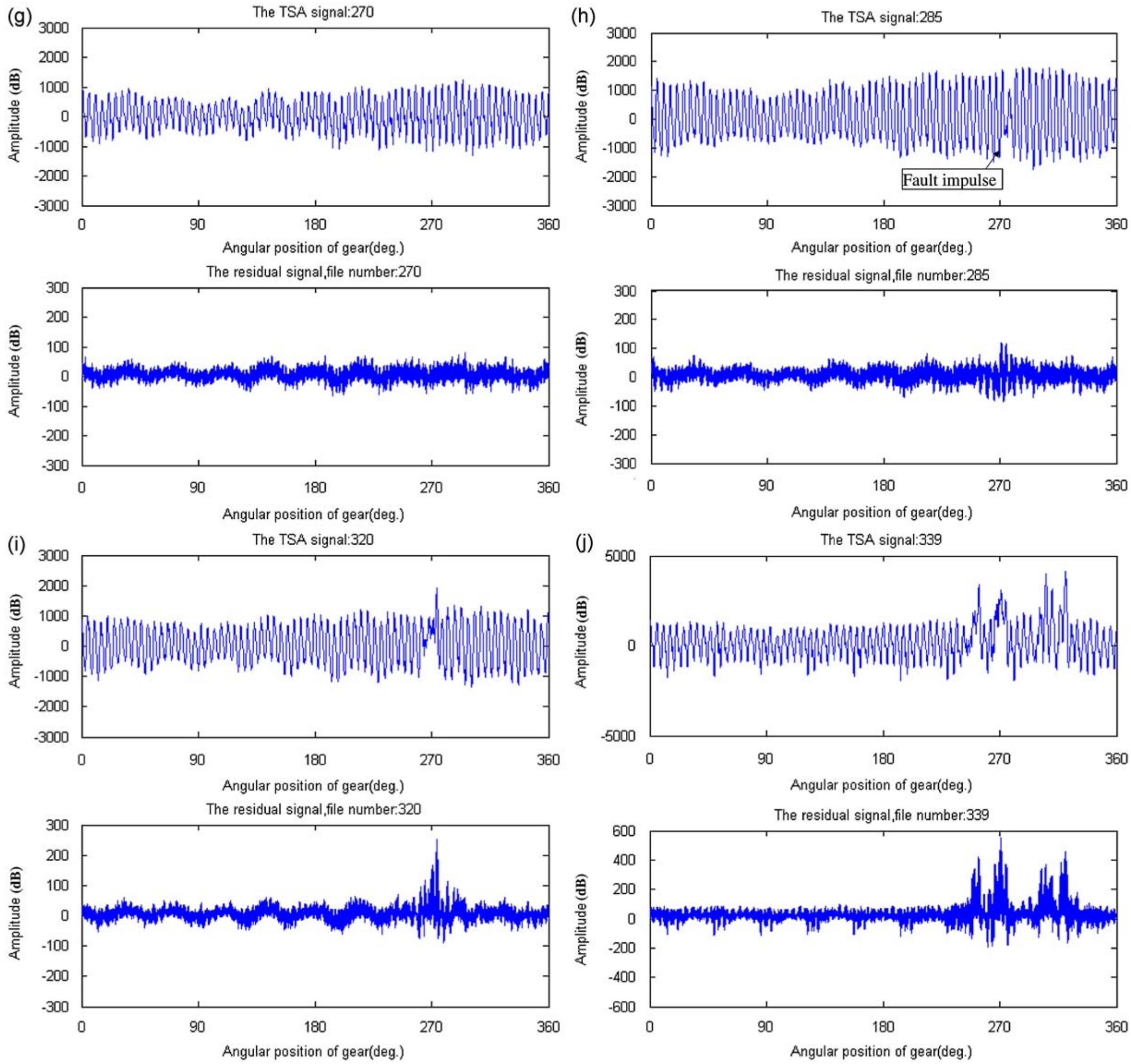

Fig. 1. (*Continued*)

The form factor and crest factor of residual signals over full gear lifetime are shown in Figs. 2(f) and (g), respectively. Although both of these statistics are insensitive to varying load and able to indicate gear damage, they are unable to diagnose early gear fault.

5.2. Continuous wavelet transform maps of residual signal

In our specific case, the analyzed frequency range is 100–3000 Hz, which includes the most important meshing harmonics (1st harmonics = 613 Hz, 2nd harmonics = 1226 Hz and 3rd harmonics = 1839 Hz). The center frequency f_c and bandwidth parameter f_b of continuous wavelet transform are experimentally determined to be 1 and 0.5 Hz. The result of the continuous wavelet transform is often graphically represented in a time-scale plane. However, using the relationship $f=f_0/a$ between frequency and scale, and by transforming the time of one wheel revolution to 360° of wheel angular location, the results of continuous wavelet transform amplitude maps are displayed in the angle-frequency plane in the following plots.

Fig. 3 shows the continuous wavelet transform amplitude maps of the time synchronously averaged signal in six running phases. The dashed lines correspond to the gear meshing frequency and to two of its harmonics. We note that most of the energy in the vibration signals is centered at the gear meshing frequency of 613 Hz. The trace shows 70 "peaks" that correspond to the 70 teeth of the gear being analyzed. In the healthy gear states (Figs. 3(a) and (b)), there is little fluctuation in trace of meshing frequency, which is realistic and due to small imperfections in the gear train. However, there is a great fluctuation in the location of 270° in the gear faulty state (Fig. 3(e)), where an energy decrease is followed by a sudden increase. It indicates that the tooth corresponding to that location has been broken. The amplitude fluctuation can be explained by the bigger impact caused by broken teeth when gearing. In Fig. 3(f), there are two great fluctuations

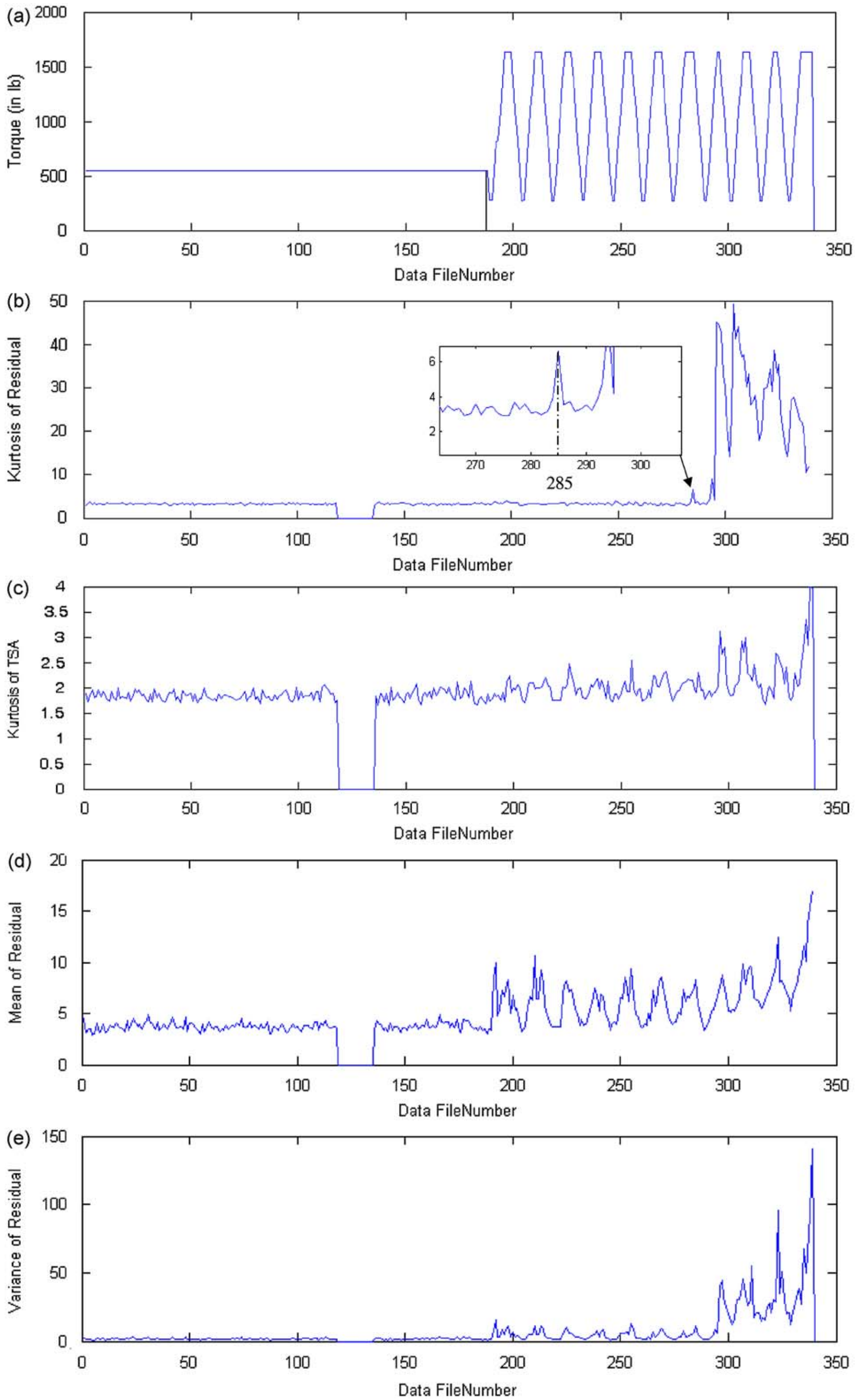

Fig. 2. Varying output torque and statistics of residual signal: (a) Varying output torque. (b) Kurtosis of residual signal. (c) Kurtosis of time synchronously averaged signal. (d) Mean of residual signal. (e) Variance of residual signal. (f) Form factor of residual signal. (g) Crest factor of residual signal.

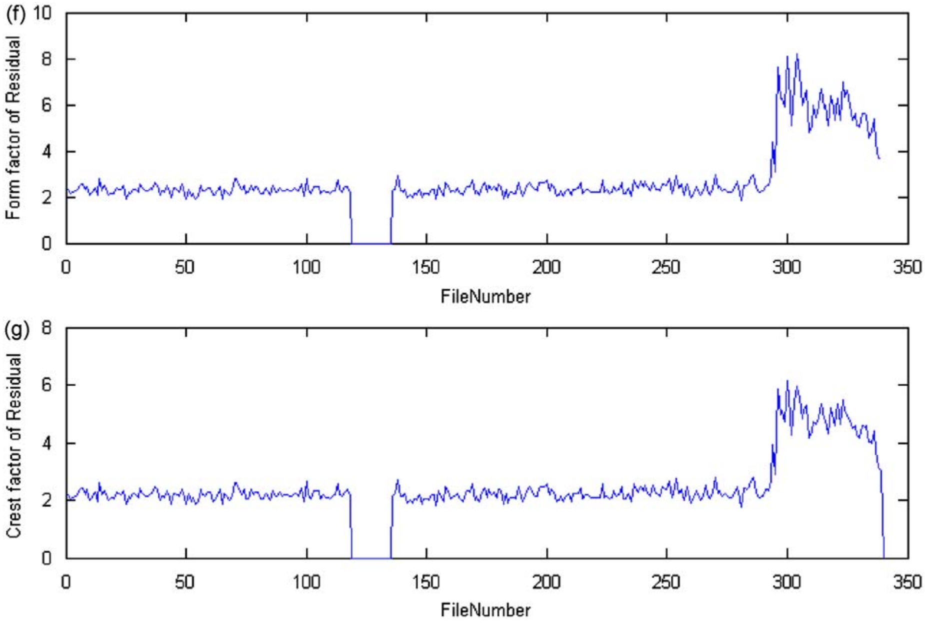

Fig. 2. (*Continued*)

around 270° and 315°, showing that several teeth have been broken. No special symptoms can be found in the map of gear small crack state (Fig. 3(d)).

Fig. 4 shows the continuous wavelet transform amplitude maps of residual signal. In residual signal, most of vibration energy generated by gear meshing action has been removed, so the amplitude peaks do not line up with the meshing frequency or any of its harmonics. No special symptoms can be found in the maps of healthy states and small crack states. However, there is also an evident fluctuation in the angular location of 270° in faulty gear state (Figs. 4(e) and (f)), which is caused by the broken teeth. Although such transient features both in Figs. 3 and 4 make it possible to localize the gear damage, these maps cannot be used to indicate and to prognosticate the gear fault advancement quantitatively.

It appears that the continuous wavelet transform amplitude maps of the time synchronously averaged signal (Figs. 3(e) and (f)) are more efficient for fault detection than those of residual signal (Figs. 4(e) and (f)) in the case of "broken teeth". The reason is that the energy in residual signal is lower than in time synchronously averaged signal because of the departure of meshing frequency and its harmonics. However, this is not true in the case of "small tooth crack" (Figs. 3(d) and 4(d)). Effective fault indicators should be able to detect the occurrence of early small gear crack and not to vary with the fluctuating load. It is not possible to obtain such indicator from the continuous wavelet transform amplitude maps of the time synchronously averaged signal. As discussed in the following section, the proposed gear fault growth parameter (FGP) obtained from the continuous wavelet transform amplitude maps of residual signal can accomplish that.

5.3. *FGP based on continuous wavelet transform amplitude of residual signal*

Fig. 5 shows the cross-section amplitude at randomly selected frequency 1600 Hz (top plot of each file) and the mean amplitude (bottom plot) in residual signal continuous wavelet transform amplitude maps which are shown in Fig. 4 under different torque levels and in different gear states.

(1) Even in the same gear healthy state, the waveform and amplitude value of continuous wavelet transform are different if the torque levels are different (Figs. 5(a)–(d)), just like the amplitude of residual signal. Hence, the continuous wavelet transform amplitude value of residual signal is inadequate for indicating fault advancement under varying load condition.
(2) It can be observed from Figs. 5(g) and (h) that continuous wavelet transform amplitude has an evident increase in gear early fault state (small crack), which is not caused by load fluctuation because the load remains constant in one data file. This phenomenon forms the basis of the fault growth parameter proposed in Section 3. In gear broken states, there are several more evident peaks of which are shown in Figs. 5(i) and (j).

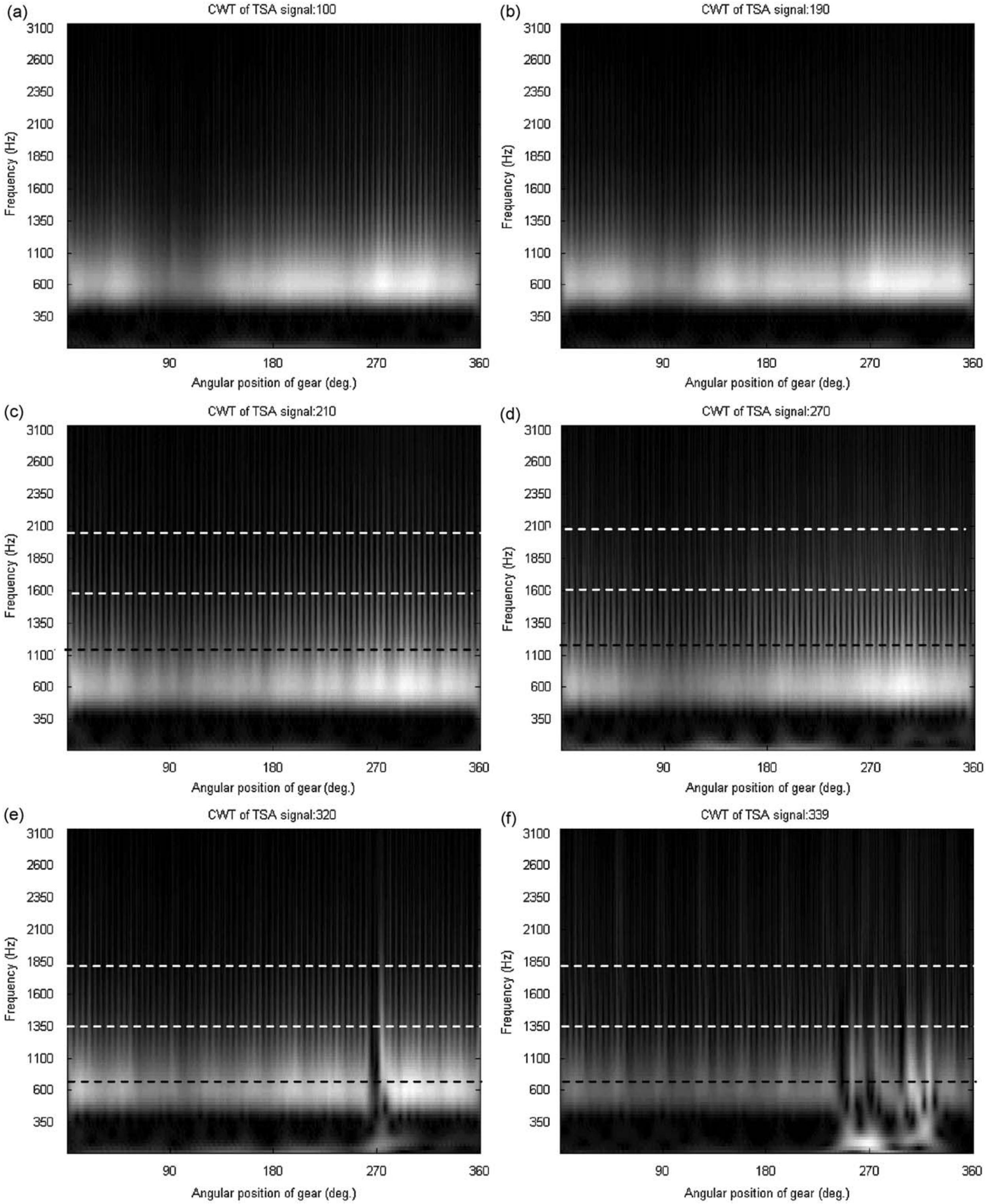

Fig. 3. Continuous wavelet transform amplitude maps of time synchronously averaged signal: (a) 100% nominal torque level (62.28 N-M), healthy state. (b) 50% nominal torque level (31.31 N-M), healthy state. (c) 300% nominal torque level (184.46 N-M), healthy state. (d) 200% nominal torque level (123.77 N-M), teeth crack. (e) 200% nominal torque level (123.77 N-M), teeth broken. (f) 300% nominal torque level (184.46 N-M), teeth broken.

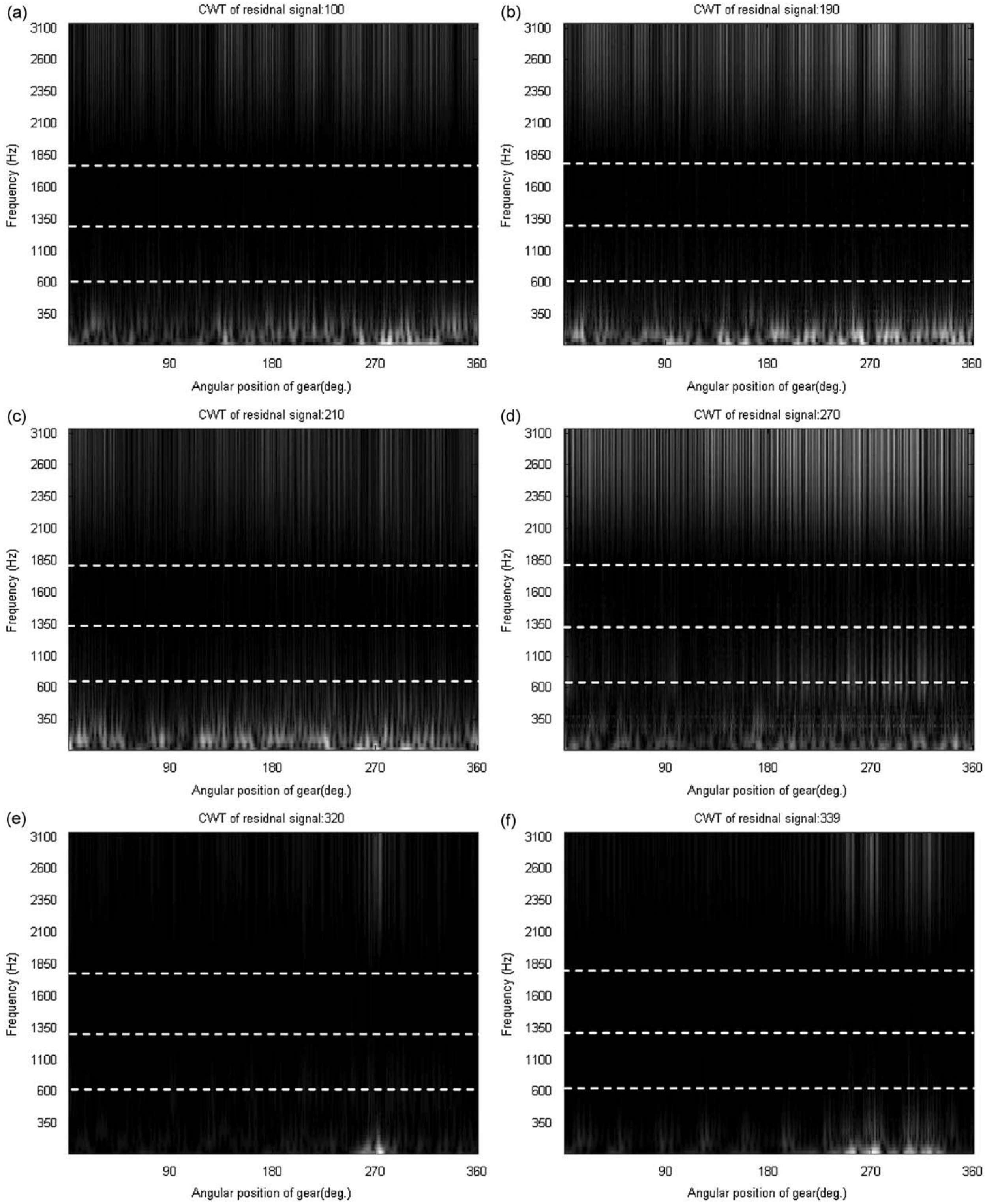

Fig. 4. Continuous wavelet transform amplitude maps of residual signal: (a) 100% nominal torque level (62.28 N-M), healthy state. (b) 50%nominal torque level (31.31 N-M), healthy state. (c) 300% nominal torque level (184.46 N-M), healthy state. (d) 200% nominal torque level (123.77 N-M), tooth crack. (e) 200% nominal torque level (123.77 N-M), teeth broken. (f) 300% nominal torque level (184.46 N-M), teeth broken.

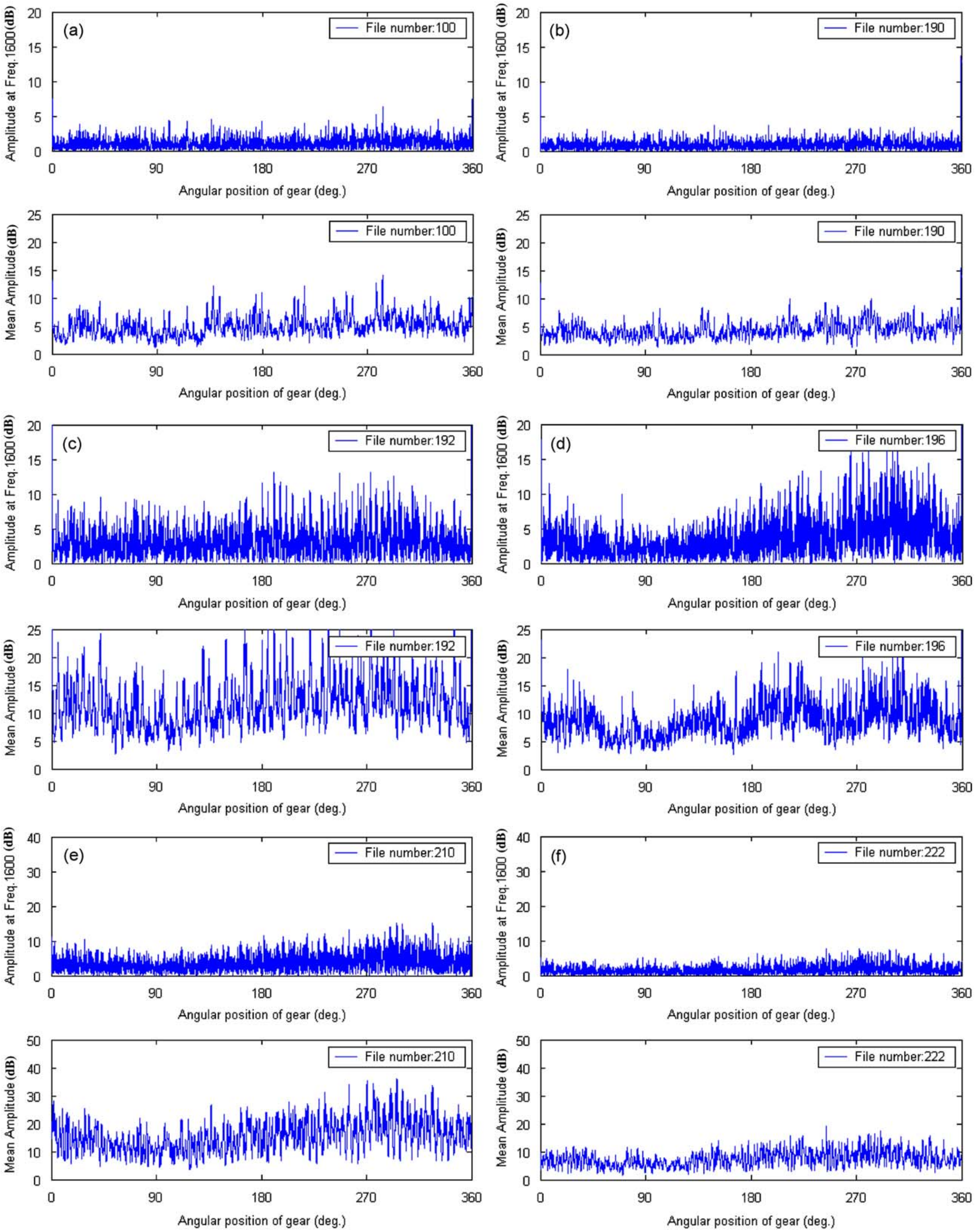

Fig. 5. Cross-section and mean continuous wavelet transform amplitude of residual signal: (a) 100% nominal torque level (62.28 N-M), healthy state. (b) 50% nominal torque level (31.31 N-M), healthy state. (c) 300% nominal torque level (184.46 N-M), healthy state. (d) 150% nominal torque level (92.12 N-M), healthy state. (e) 300% nominal torque level (184.46 N-M), healthy state. (f) 200% nominal torque level (123.77 N-M), healthy state. (g) 200% nominal torque level (123.77 N-M), tooth crack. (h) 250% nominal torque level (154.51 N-M), teeth broken. (i) 200% nominal torque level (123.77 N-M), teeth broken. (j) 300% nominal torque level (184.58 N-M), teeth broken.

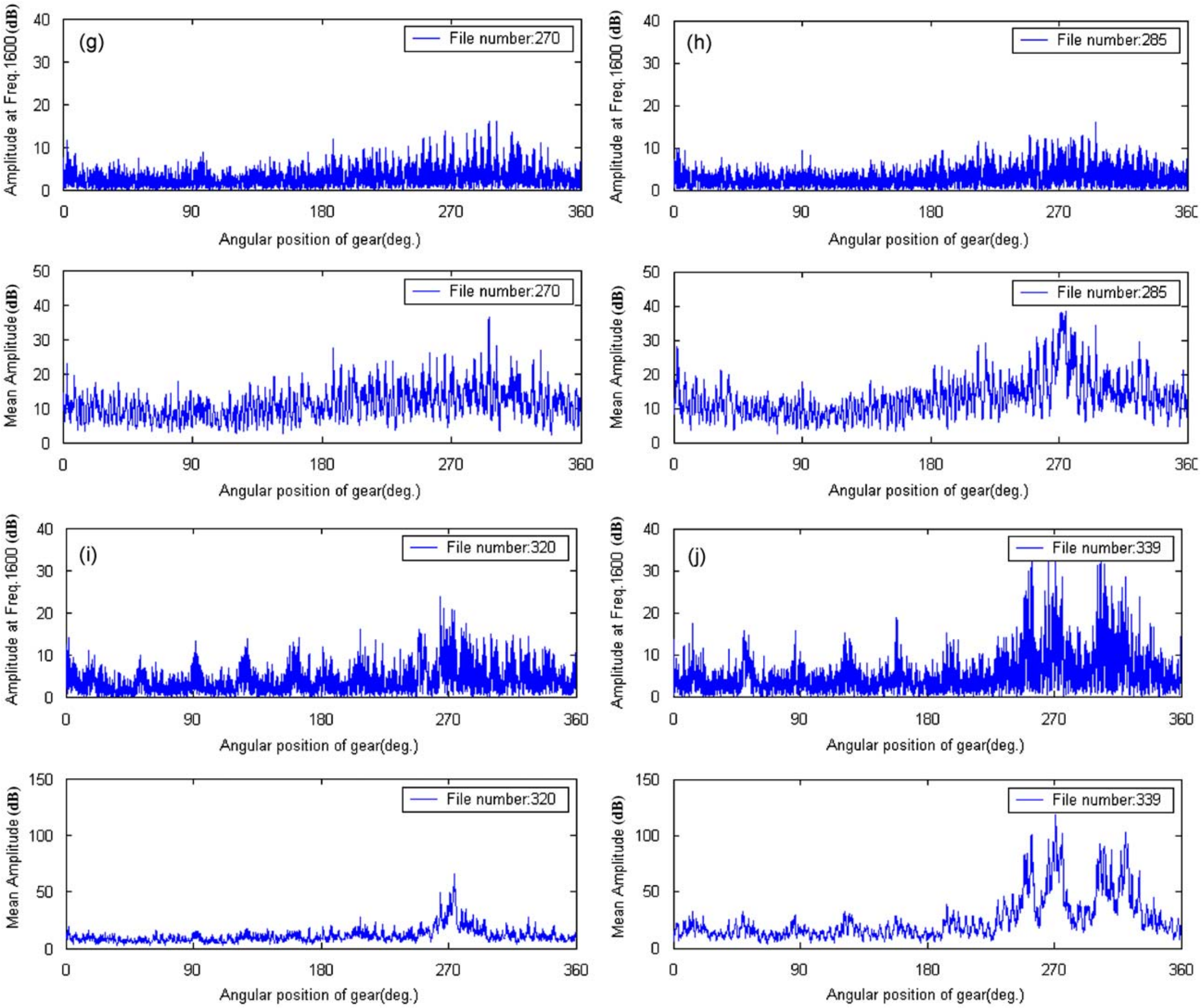

Fig. 5. (*Continued*)

Fig. 6(b) shows the FGP based on the continuous wavelet transform amplitude of residual signal *vs.* data file number over the full lifetime of gearbox.

(1) FGP is insensitive to varying load. Before data file 188, the torque level (100% nominal) is constant, and the FGP value is also constant (1.948). The gear is in healthy state. Between data files 188 and 270, the torque varies sinusoidally (Fig. 6(a)), however, the FGP maintains approximately the same constant value (1.948) and the gear is also in healthy state.
(2) FGP is sensitive to early gear fault. In data file 270, there is an evident increase in FGP value, revealing that an early damage (small crack) has occurred in gear teeth. This indication can give us an early fault alarm to examine gearbox and take some preventive actions. There is an evident improvement in comparison with the residual signal statistics shown in Fig. 2.

In order to evaluate the occurrence and advancement of gear failure quantitatively, the mean FGP before data file 270, $\overline{FGP}$, is calculated, and the percent ratio of FGP to $\overline{FGP}$ is also obtained. Table 2 shows the results. The varying of FGP is independent of torque level. In general, the FGP value has an increasing trend, so FGP can be used as a fault growth indicator, although it decreases in some data files. In practice, we can set an alarm of FGP percent threshold (such as 150%) for gear state monitoring. The time when FGP is greater than the threshold value can be set as the gearbox checking time.

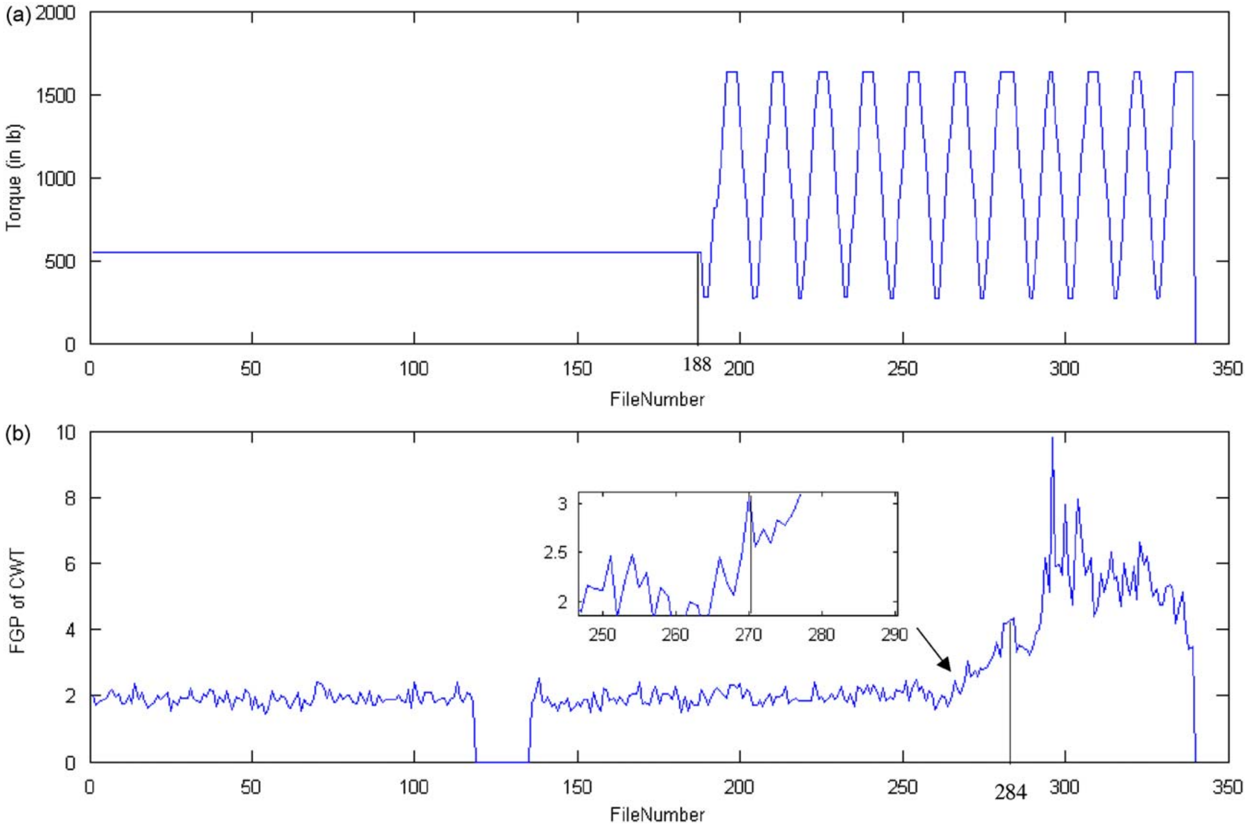

Fig. 6. FGP of continuous wavelet transform amplitude and torque level: (a) Varying output torque. (b) FGP of continuous wavelet transform.

Table 2
FGP value and torque level.

Data file number	Torque (N-M)	FGP	$FGP/\overline{FGP}*100\%$	Data file number	Torque (N-M)	FGP	$FGP/\overline{FGP}*100\%$
270	154.39	3.0714903	157.66	290	30.97	3.4367381	176.40
271	123.77	2.5625910	131.54	291	62.62	3.9049296	200.44
272	93.14	2.7409530	140.70	292	93.14	4.0256164	206.63
273	62.17	2.5927487	133.08	293	123.77	4.6821565	240.33
274	30.29	2.8347483	145.51	294	154.29	6.1702201	316.71
275	30.74	2.7795175	142.67	295	184.46	5.1185584	262.73
276	60.89	2.8906289	148.37	296	184.24	9.7839095	502.20
277	93.14	3.0884390	158.53	297	154.17	5.9240980	304.08
278	123.77	3.2534576	166.00	298	123.77	5.9316066	304.46
279	154.29	3.6279995	186.22	299	93.14	5.6496514	289.99
280	184.46	3.1640560	162.41	300	62.17	7.7556770	398.09
281	184.46	4.1640560	213.73	301	30.33	5.7946081	297.43
282	184.58	4.1640560	213.74	302	30.33	4.7368958	243.14
283	184.58	4.3140560	221.44	303	62.28	6.5397508	335.68
284	184.58	4.3640560	224.00	304	93.14	7.9455231	407.84
285	154.51	3.3073569	169.76	305	124.11	6.9485323	356.66
286	123.77	3.5451749	181.98	306	154.28	5.6611072	290.58
287	93.14	3.4381444	176.48	307	184.35	5.8512250	300.34
288	62.17	3.3958646	174.31	308	184.46	6.1659850	316.50
289	30.77	3.2347219	166.04	309	184.58	4.4148741	226.61

$\overline{FGP} = 1.9482109$.

In order to compare with the behavior of FGP, the form factor, the kurtosis and crest factor of mean amplitude of continuous wavelet transform maps over gear full lifetime are shown in Figs. 7(a)–(c), respectively. Although these statistics are insensitive to varying load and can indicate serious gear damage, they are unable to detect an early gear fault, just like the residual signal. The mean and variance of mean amplitude of continuous wavelet transform maps over gear

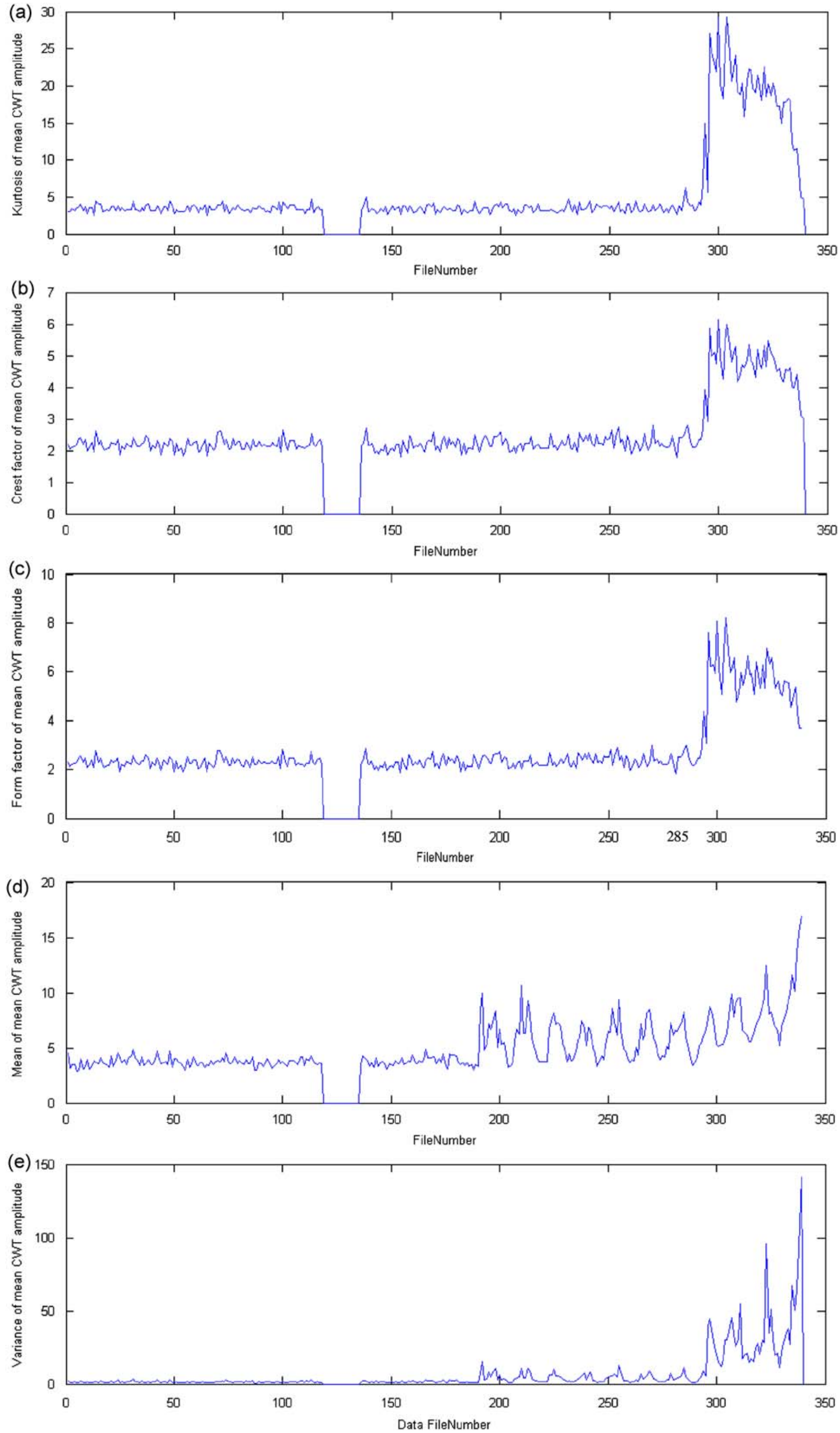

Fig. 7. Statistics of continuous wavelet transform amplitude: (a) Kurtosis of mean continuous wavelet transform amplitude. (b) Crest factor of mean continuous wavelet transform amplitude. (c) Form factor of mean continuous wavelet transform amplitude. (d) Mean of mean continuous wavelet transform amplitude. (e) Variance of mean continuous wavelet transform amplitude.

full lifetime are plotted in Figs. 7(d) and (e), respectively. The figures show that both of these statistics are sensitive to varying load.

6. Conclusions

(1) Gear motion residual signal represents the departure of time synchronously averaged signal from the average tooth-meshing vibration and usually shows evidence of faults and is much less sensitive to the alternating load condition, but the influence of fluctuating load in residual signal cannot be ultimately eliminated only by removing the fundamental and several harmonics of the tooth-meshing frequency from the FFT spectrum of a time synchronously averaged signal. Although kurtosis of residual signal is sensitive to serious gear damage and insensitive to varying load, it is not proportional to the advancement of gear fault.
(2) Continuous wavelet transform map of residual signal can localize the gear damage effectively but it is unable to evaluate gear advancement quantitatively. The proposed fault growth parameter (FGP), which is based on the continuous wavelet transform amplitudes over all transform scales, measures the relative amplitude change. FGP is not affected by varying loads in residual signal, and it can indicate gear fault advancement effectively even in an early gear fault stage.
(3) The statistics such as kurtosis, mean, variance, form factor and crest factor, both of residual signal and mean continuous wavelet transform amplitude are either sensitive to load or insensitive to early gear faults, so they cannot be used effectively as indicators of gear fault growth under varying load condition.

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